Scientific Research & Development

THE TETRAHEDRON AS A MATHEMATICAL ANALYTIC MACHINE

By: Ramón Aguilar-Achá (*)

INTRODUCTION

The unceasing advancement of science and technology compel to scientists and researchers in mathematics to encourage them in new and creative approaches to deal with the difficult problems not yet solved in the third millennium, as: 1) the Law of primes, 2) the complex problem of factorization, and 3) The Riemann hypothesis.

BACKGROUND

It is known that mathematics is accumulative in its knowledge, and throughout the centuries this science, thanks to the efforts of great thinkers, creators, discoverers, and professional and amateur mathematicians, has reached important advancements in its search for the unveiling of the inner truths and the secrets, such as the knowledge of the rule, pattern or Law of the Primes.

METHODOLOGY

The need of an alternative approach, but truly scientific, to solve this problematic situation related to the theory of pure and applied mathematical complexity; or more appropriately, the analytic number theory, of our interest, has directed us to examine new methods and techniques, looking for the solution of what we call "The Decoding of the Law of Prime Numbers"

THE MODEL

In strict sense, we depart from one of Euler's theorem and also use some other fundamental properties of the complex functions. Our model will be the **tetrahedron**, as was revealed to us through the astronomical observation of the **"Southern Cross"**.

Why the tetrahedron? Because it is the most elemental regular polyhedron of the 3-D Geometry, as a solid limited by four plane surfaces, faces or polygons, four angles and six sides totally congruent. Congruent faces are opposed by congruent angles and vice versa. To ensure - in our definition - that the four triangles of our tetrahedron are always congruent between them, in general, it is sufficient to compare three of its elements in our criterion, where one of them must be a side.

Under this definition, which relates the number of faces with the sides and where in each vertex (angle) meet the same number of faces, we state that $\varphi = 3$ because the faces are triangles, and $\varepsilon = 3$ the vertex or point where three faces meet, then we state that *v*, *f*, and *s* represent the number of vertex (corners), faces, and sides of the tetrahedron.

Since, according to a well known theorem of Euler, blowing up the tetrahedron into a sphere (in a topological sense) it is fully proved that the same could be applied to polyhedrons and maps, he generates with his discovery, the formula:

$$v + f = s + 2 \qquad (1)$$

which relates the number of corners, faces and sides, according to the algebraic signs as positive integers applied to our tetrahedron, as generated directly form Euler's theorem. Consequently, by the formula, in numbers we get:

$$4 + 4 = 6 + 2$$

or

$$8 = 8$$

With total explicit solution

φ	ε	*f*	*s*	*v*
3	3	4	6	4

Then, to our regular tetrahedron, the most elemental of all polyhedrons, some call the first platonic or mystic solid, together with the other four regular polyhedrons: the hexahedron, octahedron, dodecahedron and the icosahedron, in total five and only five regular polyhedrons, by purely geometrical reasons, as *necessary* and *sufficient* conditions for their construction.

The same can be proved in relation to the construction of our tetrahedron and the four other regular polyhedrons, according to the restricted definition of Euclid: as triangles whose faces are regular and congruent planes, employing concepts, methods and theorems of the metric and analytic geometries where the lengths and 60º exact angles between the sides of each of the equilateral triangles, equal dihedron and polyhedrons, in a certain manner, coincide with the definition of Euclid, which comes from the classic period (see Book XII of its *Elements*).

APPLICATIONS

Now, we can apply our **Model of the Tetrahedron** to the mathematical problem of interest, the prime numbers. We take two approaches of study:

1.- Idealize a great virtual tetrahedron or hyper tetrahedron, as an elemental but powerful abacus, where all numbers (abstract elements) of set N = {1, 2, 3, 4, 5, 6, …, n, …∞} fit as ordered spheres both cardinally and ordinally.

Consequently, the subset P = {2, 3, 5, 7, 11, 13, …, p, … ∞} of the ordinary or absolute primes also fit in our abacus with total order on each face of the virtual tetrahedron.

Interestingly, in this geometrical figure we can also study the set Z = {…, -3, -2, -1, 0, 1 2, 3, … ∞ } taking two adjacent congruent faces with a vertex on the top.

The study of set Q = {m/n } where m and are integers n ≠ 0, etc. as ascending or descending reasons or fractions of our virtual numbers taking any two different rows of spheres (integer or natural numbers) at a time. By approximate mathematical thinking and reasoning it is also possible to work with set Q' of the irrationals.

Finally, it is also possible to work with set R of the real taking each number equivalent to a point and only a point of the infinite real line on any one of the sides. The same is possible with set C of the complex numbers.

2.- Design the model of the tetrahedron with only ten spheres which represent digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 in three stages: one sphere in the first floor, three in the second and six in the third, representing the decimal numeration system. The irreducible system so built lets us, once appropriately codified, face the study of the extremely difficult prime numbers in relation to their density, distribution and structure, in a creative and really surprising manner.

CONCLUSION

The theory of numbers, or more precisely, the analytic number theory reaches in this way a new general and synthetic visualization and understanding within the framework of our tetrahedron, importantly, unifying the four branches of the mathematical science: Geometry, Arithmetic, Algebra and Analysis.

It is, then, possible to test a finished, logic and structured theory, and attack simultaneously, among others, a) the density, b) the distribution, c) the structure of the prime numbers. We can also deal with hard problems as the crucial two related to: 1) The Riemann hypothesis, and 2) the factorization problem of huge numbers.

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Additional Information e-mail raguilar90@yahoo.es

(*) Ramón Aguilar-Achá is B.Sc., M.Sc., Ph.Dc. and bolivian scientific investigator.